

Capital Market Frictions and the Business Cycle

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Abstract

We augment a RBC model with capital and labor market frictions. We follow the approach of Wasmer and Weil (2004) which model market imperfections as search processes: firms must sequentially find a match with a bank first and then with a worker in order to start production. We show that the interactions between labor and capital market frictions may generate a financial accelerator or decelerator, depending on a parameter condition. We compare our model with US National Accounts data and with the empirical findings of Dell'Ariccia and Garibaldi (2005): we find that the financial accelerator as well as real wage rigidities help in improving the statistical properties of the model.

Keywords: capital market frictions, business cycle, financial accelerator, real wage rigidity

JEL classification: E13, E22, E24, E32

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1 Introduction

It is a well recognized fact that the labor market is characterized by imperfections: for instance, searching for workers is costly and may take time, just as searching for jobs is. Merz (1995), Andolfatto (1996) or den Haan, Ramey, and Watson (2000) nest this type of search frictions *à la* Pissarides (2000) into a real business cycle (hereafter RBC) model and study their cyclical implications. There also exist capital market imperfections: several empirical papers conclude that it is difficult for some firms to find capital; in particular Fahrner and Simon (1994) and Blanchflower (2004) emphasize that entrepreneurship and employment creation may be constrained or delayed by lack of immediately available capital. This points at localization problems in the search for capital and explores the interaction between the labor markets and the credit markets imperfections.

Dell’Ariccia and Garibaldi (2005) apply the methodology of Davis, Haltiwanger, and Schuh (1996) to bank balance sheet data and show that gross credit flows have a much more volatile behavior than net credit flows; along each cycle it occurs a massive reallocation of credit, as measured by the difference between the sum of gross flows (contraction and expansion) and their difference. The standard credit friction models only focus on net positions - see for instance Bernanke, Gertler, and Gilchrist (1999) - and cannot easily capture these movements. Matching models could instead be useful to explain stylized facts in the credit markets since they naturally allow to disentangle gross credit expansion from credit contraction flows; modeled respectively as new matches on credit and destruction of existing matches.

In this paper, we introduce both labor and capital market frictions into a dynamic general equilibrium setup and we look at their effects on the model statistical properties. We build on Wasmer and Weil (2004) - hereafter WW - and introduce double search process to account for credit and labor market imperfections. First firms match with banks in the capital market, then they look up for a worker. Finally production takes place and firms pay back their loans in the form of an interest rate, which is determined by Nash bargaining.

However, the WW model is a small partial equilibrium one-job-one-firm model. And although the model is dynamic, most of their analysis is conducted at the steady state equilibrium by the means of comparative static exercises. We instead develop a dynamic general equilibrium model, in the spirit of the RBC literature, and we provide a developed statistical analysis, by studying the impact of technology shocks on the simulated economy. Moreover, in WW, the supply of credit comes out of the blue and may be infinite. We here close the model by assuming that credit is provided by households, and is therefore limited. Finally, in WW, the price of small firm output is exogenous. We here add large firms to make this price endogenous.

More precisely, in our model economy, we assume there are two types of firms:

small intermediate producers and large final producers. The first type of firms uses one unit of labor and one unit of capital to produce, and is subject to an exogenous destruction rate every period. The large type firm buys the intermediate products and adds risk-free capital to it as according to a standard Cobb-Douglas production function. Household investments are divided into two types: one is safe capital, lent directly to large firms; the second is risky capital which is lent to small intermediate firm through the intermediation of banks. Capital frictions apply in the relationship between banks and small firms, while labor frictions apply to small firms and workers.

Comparing our model with a set-up that only entails labor market frictions, we find the following results. First, we are able to reproduce, as in an adjustment cost model, the empirical finding that consumption leads GDP over the cycle¹. Second, frictions entail amplification of the cycle in the sense of Kocherlakota (2000): one period after the shock, the level of GDP is more distant from the steady state than the size of the shock itself; moreover we find that the amplification effect is stronger the higher the matching frictions. A technology shock raises total factor productivity of the large firms and has two self enforcing effects on the economy. The shock raises the value of small firm products, increasing vacancy openings as in a standard matching model and it stimulates capital accumulation. An added twist of our model is that positive technology shocks can also reduce the tightness of the credit sector, fostering a further acceleration of the real activity: we show that this latter indirect effect (financial accelerator) arises only when the cost of keeping a vacancy open paid by the small firms are higher than the interest rates on deposits paid by the banks. This condition generalizes the results of Wasmer and Weil (2004) where financial acceleration always arises: since there are no households deposits in their model the cost for a bank is implicitly set equal to zero and any positive vacancy opening cost for the small firms does the job. If our condition does not hold (*i.e.* if we have very low vacancy costs), then we do not generate a financial accelerator but instead a financial decelerator.

Our last point is to compare simulated gross credit expansion (*i.e.* new matches in the credit market every period) moments with the empirical findings of Dell’Ariccia and Garibaldi (2005) regarding gross credit aggregates. We find that gross credit expansion volatility is dramatically increased by the introduction of real wage rigidities and that this helps the model in being more consistent with the data. The idea of real wage rigidity is also consistent with the recent debate in the labor literature, see for instance Shimer (2004).

The rest of the paper is organized as follows. We first present the model in section 2 and we give the equilibrium definition in 3. The specification and calibration of the model is discussed in 4, while the main results are in section 5, leaving some technicalities to the appendix. In section 6 we compare our simulated gross flows with the findings of Dell’Ariccia and Garibaldi (2005).

¹We thank Marcello Savioz for this observation.

Section 7 concludes.

2 The model

There are four types of agents in our economy: large representative firms, small firms, representative banks and representative households. Large firms produce final goods by the means of intermediate goods and capital; the capital for large firms is directly provided by the households, and the capital for the small firms is indirectly (through the intermediaries/banks that will bear the risk associated to the small firms) provided by the households. The final and intermediate goods markets and the large firms capital market are perfectly competitive. We assume that a small firm needs one unit of capital and one worker to be able to produce one intermediate good. Once the firm has found the capital, it searches for a worker through an imperfect labor market. Finally, when the firm produces, it pays a rent to the bank (resp. household) for the use of capital (resp. labor).

We use the Pissarides (2000) representation of the labor market, that consists of a two sided search market between firms and representative households-workers: it is difficult to locate labor supply/demand and it is costly and time consuming for a firm (resp. a worker) to search for a worker (resp. a firm). Following den Haan, Ramey, and Watson (2003) and Wasmer and Weil (2004), the financial market is modeled in a fully symmetric way, *i.e.* it consists of a two sided search market between firms and representative banks or households: it is difficult to locate capital supply/demand and it is costly and time consuming for a firm (resp. a bank) to search for a bank (resp. a firm). In both cases, the frictions may arise from information problems and geographical distance. Frictions on both financial and labor markets are represented by matching functions.

The small firm capital rent is determined as a share of the surplus generated by the production, and we assume an rigid real wage. The price of the final goods is normalized to 1, the price of the intermediate goods adjusts to clear the intermediate goods market and the interest rate adjusts to clear the capital market.

2.1 Financial and labor frictions

If L_t is the new capital supply (by the banks) and E_t the new capital demand (by the small firms), we define the number of new capital matches $H_t = \mathcal{H}(L_t, E_t)$, where \mathcal{H} is a homogeneous matching function, increasing in its arguments and satisfying constant returns to scale.

Total active population is normalized to 1 and can be employed (N_t) or unemployed ($U_t = 1 - N_t$). If V_t is the new labor demand (by the firms) and $1 - N_t$ the new excess labor supply (by the households), we define the number of new labor matches $M_t = \mathcal{M}(V_t, 1 - N_t)$, where \mathcal{M} is a matching function,

increasing in its arguments and satisfying constant returns to scale. As a result, the probability p_t^B for a bank to find a firm and the probability p_t^F for a firm to find a bank are:

$$p_t^B = \frac{\mathcal{H}(L_t, E_t)}{L_t} \quad \text{and} \quad p_t^F = \frac{\mathcal{H}(L_t, E_t)}{E_t}, \quad (1)$$

and the probability q_t^F for a firm to find a worker and the probability q_t^H for a worker to find a firm are:

$$q_t^F = \frac{\mathcal{M}(V_t, 1 - N_t)}{V_t} \quad \text{and} \quad q_t^H = \frac{\mathcal{M}(V_t, 1 - N_t)}{1 - N_t}. \quad (2)$$

The small firms can be in three different states:

- *state 1*: firms searching for a bank able to provide one unit of capital (E_t firms)
- *state 2*: firms with the unit of capital searching for one worker (V_t firms)
- *state 3*: firms with one unit of capital and one worker, producing one intermediate goods and paying back a capital rent to the bank and a labor rent (wage) to the household (N_t firms)

It is worth noting that N_t is the employment level and $V_t + N_t$ is the capital level used by small firms. If we assume that capital and labor matches are destroyed with the exogenous probability s^2 , the dynamics between the different states is:

$$V_{t+1} = (1 - s)V_t - \mathcal{M}(V_t, 1 - N_t) + \mathcal{H}(L_t, E_t), \quad (3)$$

$$N_{t+1} = (1 - s)N_t + \mathcal{M}(V_t, 1 - N_t). \quad (4)$$

2.2 Representative household

The representative household has an income that can be consumed (C_t), directly invested (I_t) into large firms or indirectly (through banks that behave as intermediaries and take risk) invested (H_t) into small firms. Its welfare satisfies the Bellmann equation:

$$W_t^H = \max_{I_t, H_t} \{ \mathcal{U}(C_t) + \beta E_t [W_{t+1}^H] \}, \quad (5)$$

under the budget constraint:

$$C_t + I_t + H_t = N_t w_t + (r_t - \delta)K_t + (r_t^b + s)(V_t + N_t) + \Pi_t^F + \Pi_t^B, \quad (6)$$

²Shimer (2005) shows on US data that the separation probability is nearly acyclical, particularly during the last decades. Hall (2005) also emphasizes that for the past 50 years in the US, the separation rate is nearly constant while the job-finding rate shows high volatility at business cycle.

and the investment definitions:

$$I_t = K_{t+1} - (1 - \delta)K_t \quad (7)$$

$$H_t = (V_{t+1} + N_{t+1}) - (1 - s)(V_t + N_t). \quad (8)$$

\mathcal{U} is an increasing and concave utility function and β is the household discount factor. Income includes wages w_t (paid by the small firms), interest r_t on capital in large firms and interest r_t^b on capital in small firms (paid by the banks). δ is the large firms capital depreciation rate and s is the small firms capital depreciation rate (equivalent to the job destruction rate). We assume that the households hold the small firms and the banks, and therefore receive their whole profits, respectively Π_t^F from the firms and Π_t^B from the banks. Maximizing (5) with respect to I_t and H_t , under the constraints (6), (7) and (8) gives:

$$\frac{\partial \mathcal{U}(C_t)}{\partial C_t} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \frac{\partial \mathcal{U}(C_{t+1})}{\partial C_{t+1}} \right], \quad (9)$$

$$\frac{\partial \mathcal{U}(C_t)}{\partial C_t} = \beta \mathbb{E}_t \left[(1 + r_{t+1}^b) \frac{\partial \mathcal{U}(C_{t+1})}{\partial C_{t+1}} \right]. \quad (10)$$

2.3 Representative bank

The representative bank decides the level of investment L_t it wants to supply, and collects the resulting number of new capital matches H_t from the households. The bank receives income ρ_t from the producing firms (N_t firms) but have to pay interests to households on the whole capital stock ($V_t + N_t$ units of capital). The representative bank's asset value satisfies the Bellmann equation:

$$W_t^B = \max_{L_t} \left\{ \rho_t N_t - k L_t - (r_t^b + s)(V_t + N_t) + \tilde{\beta}_t \mathbb{E}_t [W_{t+1}^B] \right\}, \quad (11)$$

under the flow constraints:

$$V_{t+1} = (1 - s - q_t^F) V_t + p_t^B L_t, \quad (12)$$

$$N_{t+1} = (1 - s) N_t + q_t^F V_t, \quad (13)$$

where k is the bank cost of searching for a firm (capital supply cost). Maximizing (11) with respect to L_t and under the constraints (12) and (13) gives:

$$k = \tilde{\beta}_t p_t^B \mathbb{E}_t \left[\frac{\partial W_{t+1}^B}{\partial V_{t+1}} \right]. \quad (14)$$

By the envelope theorem, we also have:

$$\frac{\partial W_t^B}{\partial V_t} = -(r_t^b + s) + \tilde{\beta}_t (1 - s - q_t^F) \mathbb{E}_t \left[\frac{\partial W_{t+1}^B}{\partial V_{t+1}} \right] + \tilde{\beta}_t q_t^F \mathbb{E}_t \left[\frac{\partial W_{t+1}^B}{\partial N_{t+1}} \right] \quad (15)$$

$$\frac{\partial W_t^B}{\partial N_t} = -(r_t^b + s) + \rho_t + \tilde{\beta}_t (1 - s) \mathbb{E}_t \left[\frac{\partial W_{t+1}^B}{\partial N_{t+1}} \right]. \quad (16)$$

Since the banks are held by the households, the rate at which future profits are discounted is:

$$\tilde{\beta}_t = \beta \mathbb{E}_t \left[\frac{\partial \mathcal{U}(C_{t+1})}{\partial C_{t+1}} \frac{\partial C_t}{\partial \mathcal{U}(C_t)} \right]. \quad (17)$$

2.4 Small firms

As already explained, the small firms can be in 3 states and their asset values are respectively given by:

$$W_t^{F,1} = -c + \tilde{\beta}_t \mathbb{E}_t \left[(1 - p_t^F) W_{t+1}^{F,1} + p_t^F W_{t+1}^{F,2} \right], \quad (18)$$

$$W_t^{F,2} = -\gamma + \tilde{\beta}_t \mathbb{E}_t \left[s W_{t+1}^{F,1} + (1 - q_t^F - s) W_{t+1}^{F,2} + q_t^F W_{t+1}^{F,3} \right], \quad (19)$$

$$W_t^{F,3} = -\rho_t - w_t + d_t + \tilde{\beta}_t \mathbb{E}_t \left[s W_{t+1}^{F,1} + (1 - s) W_{t+1}^{F,3} \right], \quad (20)$$

where c is the firm cost of searching for a bank (capital demand cost), γ is the firm cost of searching for a worker (labor demand cost), ρ_t is the capital price paid to the bank, w_t is the wage paid to the worker, d_t is the price of the intermediate goods and $\tilde{\beta}_t$ is the rate at which future profits are discounted. Since the small firms are held by the households, the rate at which future profits are discounted is still given by equation (17).

The free entry condition states:

$$W_t^{F,1} = 0. \quad (21)$$

2.5 Large representative firm

The representative firm's asset value satisfies the Bellmann equation:

$$W_t^{FF} = \max_{N_{t+1}, K_{t+1}} \left\{ \varepsilon_t \mathcal{F}(K_t, N_t) - d_t N_t - (r_t + \delta) K_t + \tilde{\beta}_t \mathbb{E}_t [W_{t+1}^{FF}] \right\}, \quad (22)$$

where ε_t is an aggregate productivity shock, \mathcal{F} is a production function satisfying the usual Inada conditions, N_t is the amount of intermediate goods (or equivalently the amount of small firms producing, or equivalently the amount of employed workers), K_t is the capital stock, r_t is the capital interest rate paid to the households, δ is the depreciation rate of capital. Maximizing (22) with respect to N_{t+1} and K_{t+1} gives respectively:

$$\varepsilon_t \frac{\partial \mathcal{F}(K_t, N_t)}{\partial N_t} = d_t, \quad (23)$$

$$\varepsilon_t \frac{\partial \mathcal{F}(K_t, N_t)}{\partial K_t} = r_t + \delta. \quad (24)$$

2.6 Labor and capital prices

We assume that real wages are fixed at their steady state level: this simplifying assumption is consistent with the fact that in real data, wages are mostly acyclical and have a low volatility, see for instance King and Rebelo (1999) for a US empirical evidence or Shimer (2004) for simulation results. In appendix 2, we assume a Nash bargained wage and compare the two economies (exogenous *vs.* Nash bargained wage):

$$w_t = \bar{w}. \quad (25)$$

We assume that the capital price ρ_t is Nash bargained between the bank and the small firm once the firm starts to produce³:

$$\max_{\rho_t} \left(W_t^{F,3} - W_t^{F,1} \right)^\zeta \left(\frac{\partial W_t^B}{\partial N_t} \right)^{1-\zeta}, \quad (26)$$

where $0 < \zeta < 1$ is the firm's bargaining power. Maximizing (26) with respect to ρ_t gives⁴:

$$(1 - \zeta) \left(\frac{\partial W_t^B}{\partial N_t} + W_t^{F,3} - W_t^{F,1} \right) = \frac{\partial W_t^B}{\partial N_t}. \quad (27)$$

3 Equilibrium definition

Given initial conditions on V_t , N_t and K_t , an equilibrium of this economy is a sequence of prices $\{\mathcal{P}_t\}_{t=0}^\infty = \{r_t, r_t^b, d_t, w_t, \rho_t\}_{t=0}^\infty$ and a sequence of quantities $\{\mathcal{Q}_t\}_{t=0}^\infty = \{C_t, L_t, E_t, V_{t+1}, N_{t+1}, K_{t+1}\}_{t=0}^\infty$ such that:

- given a sequence of prices $\{\mathcal{P}_t\}_{t=0}^\infty$, $\{C_t\}_{t=0}^\infty$ is solution to the household problem (9)
- given a sequence of prices $\{\mathcal{P}_t\}_{t=0}^\infty$, $\{L_t\}_{t=0}^\infty$ is solution to the bank problem (14)
- given a sequence of prices $\{\mathcal{P}_t\}_{t=0}^\infty$, $\{E_t, V_{t+1}\}_{t=0}^\infty$ are solutions to the small firm problem (21) and the accumulation equation (3)
- given a sequence of prices $\{\mathcal{P}_t\}_{t=0}^\infty$, $\{N_{t+1}, K_{t+1}\}_{t=0}^\infty$ are solutions to accumulation equation (4) and the large firm problem (24)
- given a sequence of quantities $\{\mathcal{Q}_t\}_{t=0}^\infty$, $\{\mathcal{P}_t\}_{t=0}^\infty$ clears the final goods market:

$$Y_t = \gamma V_t + k L_t + C_t + I_t + H_t,$$

the intermediate goods market (23) and satisfies the parity condition (10)

³We transpose to the financial market a usual bargaining rule for the labor market.

⁴Under certain circumstances (too high wages combined with strongly negative shocks), firms and banks could want to destroy the job. It is easy to check *ex post* that it never happens (see appendix 3).

- wages and small firms capital prices are set according to determination mechanisms (25) and (27)

A resume of the model equations and of the market clearing conditions is given in appendix 1.

4 Specification and calibration

We calibrate our model on quarterly data to reproduce some stylized facts for the US economy. We choose a logarithmic utility function for the household and we set β equal to 0.99, implying a steady state real interest rate of roughly 4% per year.

Large firms production technology is assumed to be Cobb Douglas:

$$Y_t = \varepsilon_t K_t^\mu N_t^{1-\mu}.$$

The capital share in production (μ) is set to 0.33. The depreciation rate of capital δ is set to 0.025 and implies a steady state capital-production ratio of 9.

For the matching functions we follow den Haan, Ramey, and Watson (2000):

$$\mathcal{H}(L_t, E_t) = \frac{L_t E_t}{(L_t^h + E_t^h)^{\frac{1}{h}}}, \quad (28)$$

$$\mathcal{M}(V_t, 1 - N_t) = \frac{V_t(1 - N_t)}{(V_t^m + (1 - N_t)^m)^{\frac{1}{m}}}, \quad (29)$$

where (i) the parameter $0 < h < \infty$ represents the matching efficiency (or the inverse of the friction level): if $h \rightarrow 0$ we have $H_t = 0$ (infinite frictions), if $h \rightarrow \infty$ we have $H_t = \min(L_t, E_t)$ (no frictions) and (ii) differently from the standard Cobb-Douglas formulation, whatever the friction level, we always have probabilities $0 < p_t^B, p_t^F < 1^5$.

Using the definitions of labor market tightness $\theta_t = V_t/U_t$ and of credit market tightness $\phi_t = E_t/L_t$, we define the following functions:

$$g(\theta_t) \equiv \frac{1}{(1 + \theta_t^m)^{\frac{1}{m}}} \quad \text{and} \quad f(\phi_t) = \frac{1}{(1 + \phi_t^h)^{\frac{1}{h}}}, \quad (30)$$

this allows us to simply rewrite the probabilities:

$$p_t^B = \phi_t f(\phi_t) \quad \text{and} \quad p_t^F = f(\phi_t), \quad (31)$$

$$q_t^F = g(\theta_t) \quad \text{and} \quad q_t^H = \theta_t g(\theta_t). \quad (32)$$

⁵We of course have symmetric properties for the matching function \mathcal{M} .

We get $g(0) = 1$, $g(\infty) = 0$ and $\partial g(\theta_t)/\partial \theta_t < 0$. Moreover $\lim_{m \rightarrow 0} g(\theta_t) = 0$, $\lim_{m \rightarrow \infty} g(\theta_t) = \min(1, 1/\theta_t)$ and $\partial g(\theta_t)/\partial m > 0$.

We get $\lim_{\theta_t \rightarrow 0} \theta_t g(\theta_t) = 0$, $\lim_{\theta_t \rightarrow \infty} \theta_t g(\theta_t) = 1$ and $\partial \theta_t g(\theta_t)/\partial \theta_t > 0$. Moreover $\lim_{m \rightarrow 0} \theta_t g(\theta_t) = 0$, $\lim_{m \rightarrow \infty} \theta_t g(\theta_t) = \min(1, \theta_t)$ and $\partial \theta_t g(\theta_t)/\partial m > 0$.

We of course have symmetric properties for function f . These definitions and properties will be used in the next section to illustrate the effects of the financial frictions.

We assume that the bargaining power ζ between the firm and the bank is 0.5, hence the total surplus is equally distributed. From monthly estimations by Hall (2005), we take an average (over the last 55 years) US job destruction rate of 4%. Drawing from OECD (2004) on firms survival rates, we compute an implied average destruction rate for firms entering the US market between 3 and 4 percent per quarter: therefore setting an equal destruction rate for jobs and firms not yet producing does not seem to us too much a restrictive assumption. Finally, for what regards the productivity shock we set an autocorrelation $\eta = 0.9$ and a standard deviation $\sigma_u = 0.005$.

The matching efficiencies, the capital search costs, the labor search cost and the wage $(h, m, c, k, \gamma, \bar{w})$ are chosen to reproduce the observed values for 5 variables, and one parameter constraint. We assume an unemployment rate of 5% ($1 - N = 0.05$), which is approximately the average US unemployment rate over the last 30 years (see for instance OECD data). By simplicity, we assume all the endogenous probabilities $p^F = q^F = p^B = q^H = 0.75$: on average, it takes slightly more than 1 quarter to obtain capital from a bank, to recruit a worker, to supply capital to a firm and to find a job⁶. Finally, we impose the same cost for searching a bank than for searching a worker, that is $\gamma = c$. At our steady state we obtain that tightness values are normalized as $\phi^{ss} = \theta^{ss} = 1$.

All numerical values are reproduced in table 1.

5 Results

We examine the behavior of the model taking the technology shock as the exogenous driving force. We then illustrate numerically and analytically the financial accelerator.

⁶We do not try to match more precise probabilities since few empirical data are available to estimate the financial market probabilities but this would not much affect our simulation results.

Symbol	Value	Symbol	Value
Depreciation and discount rates			
δ	0.025	β	0.99
Production function			
ε	1	μ	0.33
Matching functions			
h	2.53	m	2.53
Capital and labor prices			
ζ	0.5	\bar{w}	1.87
Search costs and job destruction rate			
k	0.66	s	0.04
γ	0.37	c	0.37

Table 1: Numerical parameter values

5.1 Cyclical properties

We simulate three different models: (i) a basic RBC model *à la* Hansen (1985) where all markets are perfectly competitive, (ii) a RBC model *à la* Merz (1995) with frictions on the labor market (model that nests labor market frictions *à la* Pissarides (2000) into the basic RBC model) and (iii) our model with frictions on both the financial and the labor markets, nesting the *à la* Wasmer and Weil (2004) into the basic RBC model). We use comparable calibrations for all models and compare results to business cycle characteristics of US data: sources and methodology are reported in appendix 4.

The simulation results as well as the US stylized facts are summarized in table 2. The basic RBC model displays some shortcomings when compared to the real data: (i) most variables are not persistent enough unless we introduce a very highly autocorrelated productivity shock, and (ii) all the variables are perfectly contemporaneous, while consumption leads and employment lags output in real data. Introducing frictions on the labor market only partly solves the problems: persistence in simulated data increases but still not enough; and although unemployment is now lagging, consumption is still coincident with output.

Our model further improves the simulated cyclical properties: consumption leads output, and unemployment and output are more persistent. Creating employment now first necessitates to obtain capital from a bank. The employment creation process is therefore still much slower and that explains why employment/output has a stronger persistence. It is worth noting that in the two models with frictions, we impose an exogenous wage. This is justified by the fact that in real data, the wage volatility is quite low, as well as the wage-output correlation (see for instance Andolfatto (1996) or King and Rebelo (1999) for empirical evidences). Because the wage adjustment is by definition reduced to zero, we have an overreaction to shocks of the unemployment level (too high relative standard deviation): the model with a Nash bargained wage and its

US economy (1970-2004)							
variables	st.dv. rel/GDP	k	cross corr. with GDP(t+k)				
			-4	-1	0	1	4
GDP	1.00		0.29	0.88	1.00		
consumption	0.80		0.10	0.69	0.85	0.89	0.49
investment	2.84		0.36	0.88	0.95	0.85	0.28
unemployment	7.44		-0.45	-0.90	-0.88	-0.75	-0.13

model without frictions							
variables	st.dv. rel/GDP	k	cross corr. with GDP(t+k)				
			-4	-1	0	1	4
GDP	1.00		0.07	0.69	1.00		
consumption	0.23		0.45	0.74	0.77	0.77	0.77
investment	3.58		-0.01	0.65	0.99	0.99	0.99
unemployment	15.28		0.05	-0.63	-0.98	-0.98	-0.98

model with labor frictions							
variables	st.dv. rel/GDP	k	cross corr. with GDP(t+k)				
			-4	-1	0	1	4
GDP	1.00		0.14	0.86	1.00		
consumption	0.28		0.50	0.78	0.83	0.78	0.77
investment	3.50		0.04	0.83	0.99	0.94	0.92
unemployment	10.8		-0.27	-0.97	-0.90	-0.73	-0.69

model with labor and financial frictions							
variables	st.dv. rel/GDP	k	cross corr. with GDP(t+k)				
			-4	-1	0	1	4
GDP	1.00		0.15	0.86	1.00		
consumption	0.27		0.45	0.72	0.77	0.79	0.74
investment	3.34		0.07	0.84	0.99	0.59	0.43
unemployment	11.6		-0.32	-0.99	-0.86	-0.59	-0.43

HP filtered quarterly data, with 1600 weight. US stylized facts: sources and methodology in appendix 3. Simulated data: $GDP \equiv C_t + I_t + H_t$, $consumption \equiv C_t$, $investment \equiv I_t + H_t$, $unemployment \equiv 1 - N_t$.

Table 2: Business cycle statistics

statistical properties are displayed in appendix 2.

5.2 Financial accelerator: numerical evidence

Figure 1 numerically illustrates that endogenous developments in the capital market act as a financial accelerator.

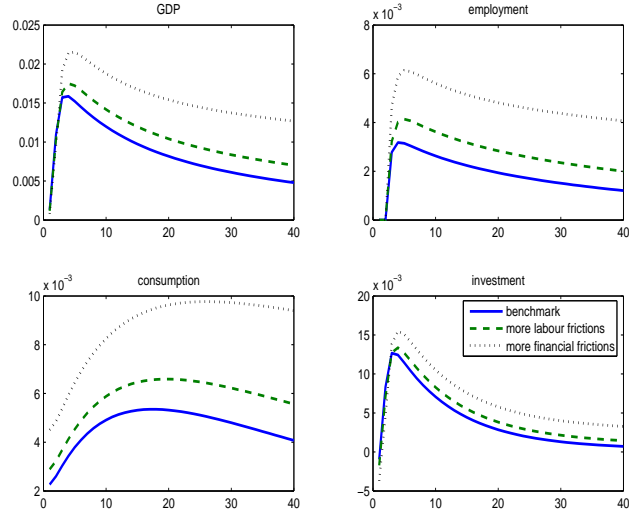


Figure 1: Impulse responses to a positive 0.5% productivity shock

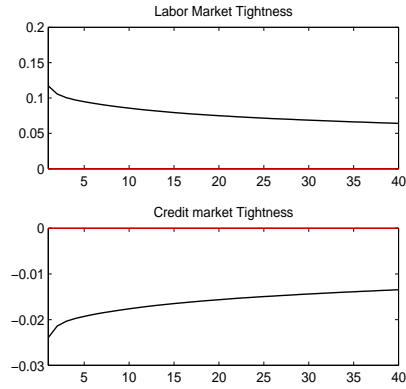


Figure 2: Impulse responses of tightness to a positive 0.5% productivity shock

A positive shock stimulates labor demand as in the standard Merz-Andolfatto setting; moreover it also raises capital supply by banks, reducing credit market tightness, as shown in figure 2. Small firms face now an increase in the probability of getting matched; this kind of positive externality increases further the small firms entry rate boosting therefore employment. The increase in labor enhances in turn capital productivity by the labor-capital complementarity link; this raises the persistence of the technological shock. A one standard deviation (0.5%) TFP shock produces an hump shaped response of output and has a stronger than 0.5% effect on output after two periods: this is the amplification effect on output as defined by Kocherlakota (2000).

In figure 1 we also compare impulse responses in an economy where there are more frictions on the labor market and in an economy where there are more frictions on the financial market⁷: both amplify the macro shock but it turns out that frictions on the financial markets are more important than frictions on the labor market to amplify the shock. The financial accelerator can be represented as the difference between the dotted lines and the dashed lines.

5.3 Financial accelerator: analytical evidence

Further investigation about this issue can be undertaken by looking at the equations describing the steady state (ϕ^*, θ^*) of the model: we show that the higher the credit market frictions the stronger the impact of (permanent) productivity shocks on the steady state. We directly consider a permanent rise in the price of intermediate goods, d ; this is equivalent to considering a permanent productivity shock since this latter triggers a permanent capital deepening, a rise in the $(K/N)_{ss}$ ratio to match the equality $1 + r_{ss} = 1/\beta$. This in turn permanently increases the d_{ss} .

As detailed in appendix 5 (proposition 1 and 2), the steady state equilibrium (ϕ^*, θ^*) is the solution of the following system of equations⁸:

$$\frac{k(s + g(\theta))}{\phi f(\phi)} + (s + r) = (1 - \zeta) \frac{g(\theta)}{s} ((d - w) - (s + r)), \quad (33)$$

$$\frac{c(s + g(\theta))}{f(\phi)} + \gamma = \zeta \frac{g(\theta)}{s} ((d - w) - (s + r)). \quad (34)$$

We also show in appendix 5 (proposition 3) under which the solution (ϕ^*, θ^*) is uniquely defined.

Equation (33) is the (θ, ϕ) equilibrium for the bank (*bank equation*). The left hand side is the cost for a bank and the right hand side is its expected income. This curve is upward sloping: if θ is low (loose labor market), the probability

⁷We decrease in turn the parameters m and h in equation (28) from 2.53 to 2.

⁸Without loss of generality we assume that $\beta = 1$.

for a firm to match a worker is high and the expected income for the bank is therefore also high, leading to high supply of capital by the bank (small ϕ).

Equation (34) represents the (θ, ϕ) equilibrium for the firm (*firm equation*). The left hand side is the cost for a firm and the right hand side is the expected income. This curve is downward sloping: if ϕ is high, there is only a few banks and the average time (cost) for a firm to find a bank is also high, which implies that a firm needs a loose labor market (low searching costs) to compensate, that is a low θ .

These two curves are plotted in figure 3. Proposition 4 (see appendix 5) shows that a positive technology shock will move both the bank curve and the firm curve to the right. As a result, θ (labor market tightness) unambiguously increases while the effect on ϕ (credit market tightness) is ambiguous.

We now try to further illustrate the financial accelerator effect. We can rewrite equations (33) and (34) as follows. By isolating $g(\theta)$ from (34) and injecting it into equation (33), we have:

$$\phi = F(d, h), \quad (35)$$

$$\theta = G(d, m, f(\phi)). \quad (36)$$

A technology shock has two effects on θ in (36): a straightforward direct one ($\partial G/\partial d > 0$) and an indirect one through ϕ : by the implicit function theorem it can be shown that $\partial G/\partial \phi < 0$. In fact we know that $\partial g(\theta)/\partial \theta < 0$ and $\partial f(\phi)/\partial \phi < 0$. Equation (34) implies that $\partial g(\theta)/\partial d < 0$ and $\partial g(\theta)/\partial f(\phi) < 0$: the previous inequalities mean that $\partial \theta/\partial d > 0$ and $\partial \theta/\partial \phi < 0$.

The accelerative (resp. decelerating) nature of the indirect effect depends on the fact that *credit market tightness* drops (resp. rises) due to the technology shock. This depends on the net cost of providing a vacancy: proposition 5 (appendix 5) shows that credit market tightness drops if the cost of bearing a vacancy by small firms (γ) is higher than the interest rate ($r + s$) paid by banks. When this is the case, banks have the incentive to expand the loan variable L more strongly than small firm entrance rate E . This causes financial market tightness ϕ to drop and amplifies (financial accelerator) the effect of productivity shock on unemployment.

The indirect effect of d on θ (through the fall in ϕ) is proportional to the financial market frictions. With a *perfect financial market*, we have $h \rightarrow \infty$ and $\lim_{h \rightarrow \infty} f(\phi) = 1^9$. The system of equations (35) and (36) are therefore rewritten as:

$$\phi = F(d), \quad (37)$$

$$\theta = G(d, m). \quad (38)$$

⁹Limit when $\phi \leq 1$, which is the case when $h \rightarrow \infty$.

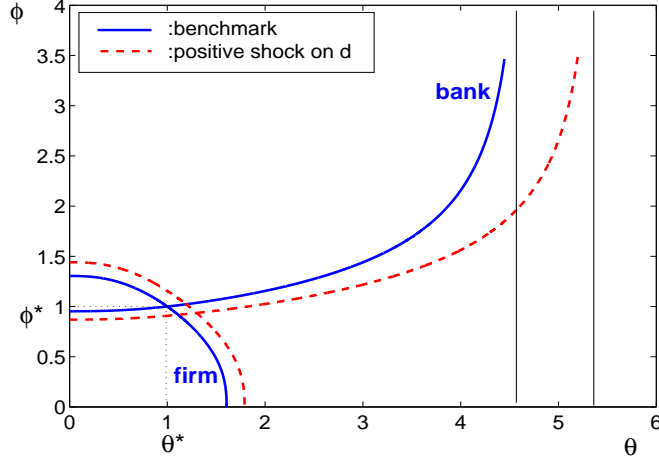


Figure 3: Equilibrium labor and financial markets: a permanent productivity shock

The indirect effect of d on θ disappears and the increase in θ following a positive productivity shock is less important. The financial accelerator (decelerator) plays through the term $f(\phi)$, which is null without financial frictions and is higher the higher the frictions. With a *perfect labor market*, we have $m \rightarrow \infty$ and $\lim_{m \rightarrow \infty} g(\theta) = 1/\theta^{10}$. The system of equations (35) and (36) remains unchanged and d has still two effects on ϕ . The financial accelerator applies even without labor market frictions.

6 Gross credit flows

Standard credit friction models only take into account net credit flows; recently Dell’Ariccia and Garibaldi (2005) have shown that focusing only on the net dimension can give a distorted representation of the credit markets. The authors apply to credit variables the methodology of Davis, Haltiwanger, and Schuh (1996): considering each bank in the dataset as a single unit they take balance sheet data and consider the net position of each unit; the sum over all sample of separate positive and negative signs gives respectively gross credit expansion and gross credit contractions. The findings of Dell’Ariccia and Garibaldi (2005) which relate to our paper are the following: gross credit expansion is more volatile than GDP by a ratio of two; gross credit expansion is procyclical but has a low contemporaneous correlation with GDP (around 0.35).

In this section we present the comparison between simulated gross credit expan-

¹⁰Limit when $\theta \geq 1$, which is the case when $m \rightarrow \infty$.

sion and the statistics reported by Dell’Ariccia and Garibaldi (2005); as a proxy for credit expansion in the model we take the number of matches in the credit market each period, h . This latter variable more tightly corresponds to the gross credit flow expansion as described in Dell’Ariccia and Garibaldi (2005); to our knowledge no data are instead available for credit market tightness moments.

Data as in Dell’Ariccia and Garibaldi (2005)							
Variable	st.dv. rel/GDP	k	cross corr. of GDP with Cred.Exp.(t+k)				
			-4	-1	0	1	4
Credit Expansion	2		-0.07	0.37	0.35	0.26	0.16
Nash bargaining wage							
Variable	st.dv. rel/GDP	k	cross corr. of GDP with Cred.Exp.(t+k)				
			-4	-1	0	1	4
Credit Expansion	0.02		-0.1	-0.1	-0.1	0.5	0.7
Rigid wages							
Variable	st.dv. rel/GDP	k	cross corr. of GDP with Cred.Exp.(t+k)				
			-4	-1	0	1	4
Credit Expansion	0.4		0.3	0.1	0.2	0.9	0.9

Table 3: Credit expansion statistics

The flexible real wage model can hardly capture credit flows dynamics; volatility of gross credit expansion is too low by many orders and it is *countercyclical*. The fixed wage version improves on both the volatility dimension and the correlations ; this is quite remarkable since no liquidity or money is explicitly in the model. Nevertheless we are not able to capture the lagging nature of gross credit flows with respect to GDP; while we are not very far from the low contemporaneous correlation between GDP and gross flows which are found in the data.

7 Conclusion

In this paper, we extend the Merz (1995) and Andolfatto (1996) RBC models with frictions on the labor market, by adding imperfections on the capital market. To do so, rather than to rely on asymmetric information and moral hazard problems as in Bernanke, Gertler, and Gilchrist (1999), we rely on localization problems as in Wasmer and Weil (2004) and model the frictions on the capital market in a fully symmetric way to the frictions on the labor market. This strategy allows us to separate credit expansion and contraction consistently with the difference in gross credit expansion and gross credit contraction found in the data by Dell’Ariccia and Garibaldi (2005)

Our main results are that (i) the introduction of the imperfect capital market improves the cyclical properties of the model (ii) the frictions on the financial

market act as a financial accelerator, that is they amplify and propagate the macroeconomic shocks. (iii) a rigid wage partially helps in reproducing credit flows statistics, even if a more complex model (with monetary shocks) are probably more suitable to match the data.

To keep this simple framework, this model is very stylized and could then be developed along several dimensions:

- Neither the exogenous wage (no flexibility at all) nor the Nash bargained wage (too much flexibility) lead to a fully convincing modelization. Making a difference between outsiders and insiders, or introducing a Calvo type wage in the search framework could improve our modelization.
- Our credit view is not related to liquidity since money is not introduced in the model. It should rather be interpreted as propensity to lend. This model could be extended by adding a monetary dimension with liquidity and a monetary policy shock.

We leave these extensions for future research.

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Appendix 1: model equations

Matching functions:

$$\mathcal{H}(L_t, E_t) = \frac{L_t E_t}{(L_t^h + E_t^h)^{\frac{1}{h}}}, \quad (39)$$

$$\mathcal{M}(V_t, 1 - N_t) = \frac{V_t(1 - N_t)}{(V_t^m + (1 - N_t)^m)^{\frac{1}{m}}}, \quad (40)$$

$$Y_t = \varepsilon_t K_t^\mu N_t^{1-\mu}, \quad (41)$$

$$p_t^B = \frac{\mathcal{H}(L_t, E_t)}{L_t} \quad \text{and} \quad p_t^F = \frac{\mathcal{H}(L_t, E_t)}{E_t}, \quad (42)$$

$$q_t^F = \frac{\mathcal{M}(V_t, 1 - N_t)}{V_t} \quad \text{and} \quad q_t^H = \frac{\mathcal{M}(V_t, 1 - N_t)}{1 - N_t}, \quad (43)$$

$$V_{t+1} = (1 - s - q_t^F)V_t + p_t^B L_t, \quad (44)$$

$$N_{t+1} = (1 - s)N_t + q_t^F V_t, \quad (45)$$

$$I_t = K_{t+1} - (1 - \delta)K_t, \quad (46)$$

$$H_t = (V_{t+1} + N_{t+1}) - (1 - s)(V_t + N_t), \quad (47)$$

$$\tilde{\beta}_t = \beta \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \right], \quad (48)$$

$$\varepsilon_t(1 - \mu)(K_t^\mu N_t^{1-\mu}) = d_t, \quad (49)$$

$$\varepsilon_t \mu(K_t^{\mu-1} N_t^{1-\mu}) = r_t + \delta. \quad (50)$$

Banks:

$$\frac{\partial W_t^B}{\partial V_t} = -(r_t^b + s) + \tilde{\beta}_t(1 - s - q_t^F)\mathbb{E}_t \left[\frac{\partial W_{t+1}^B}{\partial V_{t+1}} \right] + \tilde{\beta}_t q_t^F \mathbb{E}_t \left[\frac{\partial W_{t+1}^B}{\partial N_{t+1}} \right] \quad (51)$$

$$\frac{\partial W_t^B}{\partial N_t} = -(r_t^b + s) + \rho_t + \tilde{\beta}_t(1 - s)\mathbb{E}_t \left[\frac{\partial W_{t+1}^B}{\partial N_{t+1}} \right], \quad (52)$$

$$k = \tilde{\beta}_t p_t^B \mathbb{E}_t \left[\frac{\partial W_{t+1}^B}{\partial V_{t+1}} \right]. \quad (53)$$

Small firms:

$$c = \tilde{\beta}_t \mathbb{E}_t \left[p_t^F W_{t+1}^{F,2} \right], \quad (54)$$

$$W_t^{F,2} = -\gamma + \tilde{\beta}_t \mathbb{E}_t \left[(1 - q_t^F - s)W_{t+1}^{F,2} + q_t^F W_{t+1}^{F,3} \right], \quad (55)$$

$$W_t^{F,3} = -\rho_t - w_t + d_t + \tilde{\beta}_t E_t \left[(1-s) W_{t+1}^{F,3} \right]. \quad (56)$$

Interest rate bargaining and wages:

$$(1-\zeta) W_t^{F,3} = \zeta \frac{\partial W_t^B}{\partial N_t}, \quad (57)$$

$$w_t = \bar{w}. \quad (58)$$

An aggregate productivity shock is modeled as:

$$\varepsilon_t = \varepsilon^{1-\eta} \varepsilon_{t-1}^\eta e^{u_t}, \quad (59)$$

where η is the autoregressive parameter and $u_t \sim N(0, \sigma_u^2)$ and the goods market clear:

$$Y_t = \gamma V_t + k L_t + C_t + I_t + H_t.$$

Appendix 2: Nash bargained wage

If we assume a Nash bargained wage, equation (25) becomes:

$$\max_{w_t} \left(W_t^{F,3} - W_t^{F,1} \right)^\xi \left(\frac{\partial W_t^H}{\partial N_t} \frac{\partial C_t}{\partial \mathcal{U}(C_t)} \right)^{1-\xi}, \quad (60)$$

where

$$\frac{\partial W_t^H}{\partial N_t} = \frac{\partial \mathcal{U}(C_t)}{\partial C_t} w_t + \beta(1-s-q_t^H) E_t \left[\frac{\partial W_{t+1}^H}{\partial N_{t+1}} \right], \quad (61)$$

and $0 < \xi < 1$ is the firm's bargaining power. The maximization program gives:

$$(1-\xi) \left(\frac{\partial W_t^H}{\partial N_t} + \frac{\partial \mathcal{U}(C_t)}{\partial C_t} (W_t^{F,3} - W_t^{F,1}) \right) = \frac{\partial W_t^H}{\partial N_t}. \quad (62)$$

We simulate this model, where $\xi = 0.3$ in order to obtain a steady state wage equal to \bar{w}^{11} . Table 4 compares these simulations to our benchmark model with exogenous wage and to the empirical facts. We see that adding the bargained wage deteriorates our results, by drastically reducing the unemployment volatility and correlations.

¹¹Comparing to usual estimations (between 0.4 and 0.6) this gives a slightly too low firm bargaining power. Increasing this bargaining power would however not much affect our simulation results.

US economy (1970-2004)							
variables	st.dv. rel/GDP	k	cross corr. with GDP(t+k)				
			-4	-1	0	1	4
GDP	1.00		0.29	0.88	1.00		
consumption	0.80		0.10	0.69	0.85	0.89	0.49
investment	2.84		0.36	0.88	0.95	0.85	0.28
unemployment	7.44		-0.45	-0.90	-0.88	-0.75	-0.13

model with labor and financial frictions, exogenous wage							
variables	st.dv. rel/GDP	k	cross corr. with GDP(t+k)				
			-4	-1	0	1	4
GDP	1.00		0.15	0.86	1.00		
consumption	0.27		0.45	0.72	0.77	0.79	0.74
investment	3.34		0.07	0.84	0.99	0.59	0.43
unemployment	11.6		-0.32	-0.99	-0.86	-0.59	-0.43

model with labor and financial frictions, Nash bargained wage							
variables	st.dv. rel/GDP	k	cross corr. with GDP(t+k)				
			-4	-1	0	1	4
GDP	1.00		0.10	0.72	1.00		
consumption	0.29		0.43	0.77	0.86	0.85	0.85
investment	3.21		0.00	0.67	0.99	0.99	0.99
unemployment	0.54		-0.56	-0.69	-0.44	-0.42	-0.42

HP filtered quarterly data, with 1600 weight. US stylized facts: sources and methodology in appendix 3. Simulated data: $GDP \equiv C_t + I_t + H_t$, $consumption \equiv C_t$, $investment \equiv I_t + H_t$, $unemployment \equiv 1 - N_t$.

Table 4: Business cycle statistics

Appendix 3: Ex post check

A match (job) involves 3 agents: the worker, the firm and the bank. In *state 2*, a small firm and a bank are matched and the firm is trying to find a worker. As long as $W_t^{F,2} > W_t^{F,1} = 0$ and $\partial W_t^B / \partial V_t > 0$, both the firm and the bank have no incentive to voluntarily destroy the match.

In *state 3*, a small firm and a bank are matched through a financial market and a small firm and a worker are matched through a labor market. As long as $W_t^{F,3} - W_t^{F,1} + \partial W_t^B / \partial N_t > 0$, both the firm and the bank have no incentive to voluntarily destroy the match because the surplus to share between them is positive. The worker has no incentive to destroy the match as long as $\partial W_t^H / \partial N_t > 0$.

To avoid these endogenous job destruction problems, we check *ex post* that these 4 inequalities are never violated during our simulations.

Appendix 4: Description of the data source and methodology

US data used in the paper include GDP, Consumption, Investment and Unemployment rate. The first three series come from the Quarterly National Accounts database of the OECD, from 1970q1 to 2004q4. All data are transformed into their logarithm and HP filtered with a 1600 weight.

The last series comes from the OECD Main Economic Indicator database, from 1970m1 to 2004m12. This is a monthly series and we take the three months average value for the quarter of interest. Unemployment rate data are HP filtered with a 1600 weight.

Appendix 5: Derivation of the steady state and analytical properties

Bank equation

$$\frac{k(s + g(\theta))}{\phi f(\phi)} + (s + r) = (1 - \zeta) \frac{g(\theta)}{s} ((d - w) - (s + r)),$$

Plugging equations (16) and (20) into (27), we have

$$\rho = (1 - \zeta)(d - w) + \zeta(r + s).$$

By injecting (14) and (16) into (15), and using the definitions of the probabilities (expressions (31) and (32)) and ρ , we get equation (33).

Proposition 1

ϕ is increasing on $[0, \theta^B]$ and takes values between $[\phi^B, \infty]$.

Proof

The increasing part can be shown by the means of the implicit function theorem on equation (33). From the definitions (30), we know that $\partial\phi f(\phi)/\partial\phi > 0$ and $\partial g(\theta)/\partial\theta < 0$. From (33), we can also show that $\partial\phi f(\phi)/\partial g(\theta) < 0$. From the last 3 inequalities, we can immediately deduce $\partial\phi/\partial\theta > 0$. From equation (33) we deduce ϕ^B : if $\theta=0$ then $g(\theta) = 1$. If $\phi \rightarrow \infty$ then $\phi f(\phi) = 1$ and we can directly deduce the vertical asymptote θ^B .

QED

Firm equation

$$\frac{c(s + g(\theta))}{f(\phi)} + \gamma = \zeta \frac{g(\theta)}{s} ((d - w) - (s + r)).$$

By injecting (18), (20) into (19), and using the definitions of the probabilities (expressions (31) and (32)) and ρ , we get equation (34).

Proposition 2

ϕ is decreasing on $[0, \theta^F]$ and takes values between $[0, \phi^B]$.

Proof

The decreasing part is shown by using the implicit function theorem. From the definitions (30), we know that $\partial f(\phi)/\partial\phi < 0$ and $\partial g(\theta)/\partial\theta < 0$. From (34), we can also show that $\partial f(\phi)/\partial g(\theta) < 0$. From the last 3 inequalities, we can immediately deduce $\partial\phi/\partial\theta < 0$. The remaining part is proved as follows. We look at equation (34). If $\theta=0$ then $g(\theta) = 1$ and we can directly deduce ϕ^F . If $\phi = 0$ then $f(\phi) = 1$ and we can directly deduce θ^F .

QED

Existence and uniqueness of the steady state equilibrium

Proposition 3

If $0 < \phi^B < \phi^F$ then we have the existence of a unique and well-defined solution.

Proof

From the two previous propositions.

QED

Proposition 4

A positive productivity shock (increase in d) increases the labor market tightness (increase in θ).

Proof

From the f properties, we have $\partial\phi f(\phi)/\partial\phi > 0$. Using equation (33) with θ constant, we have $\partial\phi f(\phi)/\partial d < 0$, that is, given the previous result: $\partial\phi/\partial d < 0$. In other words, a positive productivity shock moves the bank curve to the right. From the f properties, we also have $\partial f(\phi)/\partial\phi < 0$. Using equation (34) with θ constant, we have $\partial f(\phi)/\partial d < 0$, that is, given the previous result: $\partial\phi/\partial d > 0$. In other words, a positive productivity shock moves the firm curve to the right. If the two curves are moving to the right, this will automatically give a higher θ .

QED

Proposition 5

If banks and firms have an equal bargaining power $\zeta = 1 - \zeta = 0.5$, then $\frac{d\phi^{ss}}{dd} < 0$ iff the following condition holds:

$$\gamma > (s + r).$$

Proof

When $\zeta = 1 - \zeta$, the left hand sides of the bank (33) and firm (34) equations can be equalized. After some rearrangements we have that:

$$k(s + g(\theta)) = -\phi f(\phi)(s + r) + [c(s + g(\theta)) + f(\phi)\gamma]\phi, \quad (63)$$

Now consider the system of equations (33) and (63). The bank equation (33) is upward sloping (see proposition above) and it shifts to the right with a positive technology shock. Equation (63) is independent on d . So it is sufficient (and necessary) to check that equation (63) is downward sloping in the space (ϕ, θ) to ensure that ϕ^{ss} will be negatively affected by a productivity shock. Applying the implicit function theorem to equation (63) we get:

$$\frac{d\phi}{d\theta} = - \frac{g'(\theta)(k - c\phi)}{(-\gamma + s + r)(f(\phi) + \phi f'(\phi)) - c(s + g(\theta))}. \quad (64)$$

From (63), we obtain:

$$(k - c\phi)(s + g(\theta)) = -\phi f(\phi)(s + r - \gamma).$$

By introducing this expression into (64), it is easy to show that:

$$d\phi/d\theta < 0 \iff s + r - \gamma < 0.$$

QED

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